Question 8

The tangent to the curve $y = -x^2 + 5x$ at the point where $x = 1$, crosses the x-axis at the point $(a,0)$. Find the value of a.

$\frac{dy}{dx} = -2x + 5$, $x = 1$

$\therefore$ gradient $= -2(1) + 5 = 3$

now when $x = 1$, $y = -(1)^2 + 5(1)$
$= 4$

$\therefore (1, 4)$

$\therefore$ line with
$m = 3$
$x = 1$
$y = 4$

$y - y_1 = m(x - x_1)$
$\therefore y - 4 = 3x - 3$
$\therefore y = 3x + 1$

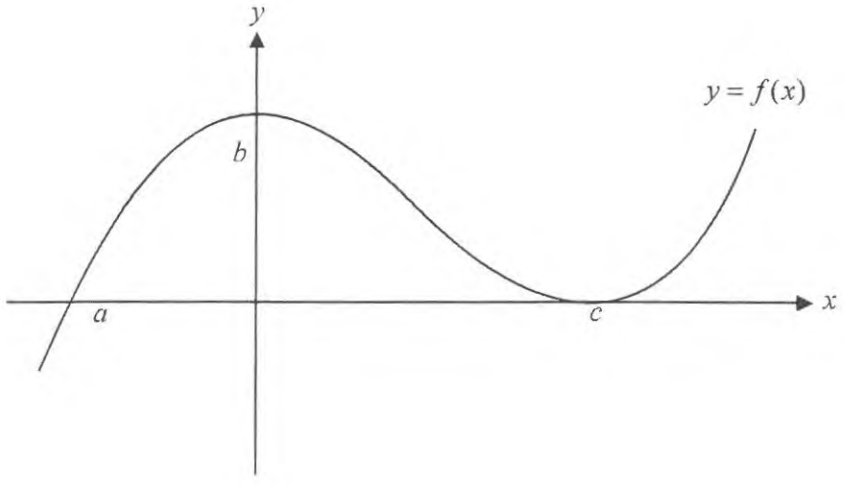
x-int $= (a, 0)$
$\therefore$ let $y = 0$
$\therefore 0 = 3x + 1$
$\therefore -1 = 3x$
$\therefore x = \frac{-1}{3}$

so $\boxed{a = \frac{1}{3}}$

4 marks

Question 9

The graph of a cubic function with rule $y = f(x)$ is shown below. There are stationary points located at $(0, b)$ and $(c, 0)$.



a. Write down the three linear factors of the function f.

$(x-a)(x-c)^2 = 0$

$\therefore$ factors are $(x-a)(x-c)$ and $(x-c)$

b. Find the values of x for which $f(x) > 0$.

function is positive → above x-axis

$\therefore$ $a < x < c$, $c < x < \infty$ or $x \in (a, \infty) \setminus \{c\}$

c. Find the values of x for which $f'(x) < 0$.

where is gradient negative?

$x \in (0, c)$

1 + 1 + 1 = 3 marks