

Measures of spread

What we already know:

- Measures of spread include:
 - Variance
 - Standard deviation
 - Range
 - Interquartile range (IQR)

Variance

The variance of the random variable X is a measure of the spread of a probability distribution about its mean. It is considered as the long-run average value of the square of the distances from the values of X to μ .

The **variance** is defined as:

$$\mathit{var}(X) = E[(X - \mu)^2] = \sigma^2$$

To calculate variance, use

$$\mathit{Var}(X) = E(X^2) - \mu^2$$

Note also: $\mathit{Var}(aX + b) = a^2 \mathit{Var}(X)$ (as for discrete variables)

The standard deviation

The standard deviation of X is a measure of spread that has the same units as the variable X .

The **standard deviation** of X is defined as:

$$\mathit{sd}(X) = \sigma = \sqrt{\mathit{var}(X)}$$



Example:

Find the variance and standard deviation of the random variable X which has the probability density function f with rule:

$$f(x) = \begin{cases} 0.5x & 0 \leq x \leq 2 \\ 0 & x < 0 \text{ or } x > 2 \end{cases}$$



Range

The **range** of a random variable is the **difference** between the smallest and largest value the variable may take. This can be determined from the domain of the probability density function.

Interquartile range

The interquartile range (IQR) is the range of the middle 50% of the distribution. It is the difference between the 75th percentile (Q_3) and the 25th percentile (Q_1).

The Interquartile range

$$IQR = Q_3 - Q_1$$



Example:

Find the range and interquartile range of the random variable X which has the probability density function f with rule:

$$f(x) = \begin{cases} 0.5x & 0 \leq x \leq 2 \\ 0 & x < 0 \text{ or } x > 2 \end{cases}$$



Example:

If X is a continuous random variable with mean $\mu=12$ and variance $\sigma^2=3$,

- Find $E(2X + 3)$
- Find $Var(3 - 5X)$



Exercises to complete

15D

