Question 3

a. Let $f(x) = e^{\cos(x)}$. Find $f'(x)$

if $f(x) = e^{g(x)}$, $f'(x) = g'(x)\, e^{g(x)}$

$g(x) = \cos(x)$

$g'(x) = -\sin(x)$

$\therefore f'(x) = -\sin(x)\, e^{\cos(x)}$

1 mark

b. Let $y = x\tan(x)$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$.

$y = u \times v$ where $u = x$, $v = \tan(x)$

$\therefore u' = 1$, $v' = \sec^2(x)$

product rule: $\frac{dy}{dx} = uv' + vu' = x\sec^2(x) + \tan(x)$

when $x = \pi/6$, $\frac{dy}{dx} = \frac{\pi}{6} \times \frac{1}{\cos^2(\pi/6)} + \tan\left(\frac{\pi}{6}\right)$

$= \frac{\pi}{6} \times \frac{4}{3} + \frac{1}{\sqrt{3}}$

$= \frac{2\pi}{9} + \frac{1}{\sqrt{3}}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\therefore \cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}$

$\therefore \frac{1}{\cos^2\left(\frac{\pi}{6}\right)} = \frac{4}{3}$

3 marks

Question 1

Let $f(x) = \frac{x^3}{\sin(x)}$. Find $f'(x)$.

$f(x) = \frac{u}{v}$ where $u = x^3$ $v = \sin(x)$

$\therefore u' = 3x^2$ $v' = \cos(x)$

Quotient rule: $f'(x) = \frac{vu' - uv'}{v^2} = \frac{3x^2\sin(x) - x^3\cos(x)}{\sin^2(x)}$

$= \frac{x^2(3\sin(x) - x\cos(x))}{\sin^2(x)}$ ← optional step - not necessary

2 marks

b. Let $g(x) = \log_e(\tan(x))$. Evaluate $g'\left(\frac{\pi}{4}\right)$.

if $g(x) = \log_e(f(x))$, $g'(x) = \frac{f'(x)}{f(x)}$

$$\left.\begin{array}{l} f(x) = \tan(x) \\ f'(x) = \sec^2(x) \end{array}\right\} g'(x) = \frac{\sec^2(x)}{\tan(x)}$$

$= \frac{1}{\cos^2(x)} \times \frac{1}{\tan(x)}$

2 marks

$= \frac{1}{\cos^2(x)} \times \frac{\cos(x)}{\sin(x)}$

$= \frac{1}{\sin(x)\cos(x)}$

so $g'\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)}$

$= \frac{1}{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}}$

$= 2$

Question 1

a. Let $y = (3x^2 - 5x)^5$. Find $\frac{dy}{dx}$.

chain rule $y = u^5$, $u = 3x^2 - 5x$

$\frac{dy}{dx} = u'y'$

$\therefore y' = 5u^4 \quad u' = 6x - 5$

$\frac{dy}{dx} = 5u^4 \times (6x-5)$

$= 5(3x^2 - 5x)^4(6x-5)$

b. Let $f(x) = xe^{3x}$. Evaluate $f'(0)$.

$u = x \quad v = e^{3x}$

$u' = 1 \quad v' = 3e^{3x}$

product rule: $f'(x) = u'v + v'u$

$= e^{3x} + 3xe^{3x}$

$\therefore f'(0) = e^{3\times 0} + \cancel{3\times 0\, e^{3\times 0}}$ 0

$= 1$

2 + 3 = 5 marks

Question 4

a. Differentiate $(x+2)\sqrt{(x-1)}$, giving your answer as a single fraction.

$= u \times v$ where $u = x+2 \quad v = (x-1)^{1/2}$ so $v = a^{1/2}$ where $a = x-1$

$\therefore v' = \frac{1}{2}a^{-1/2} \quad a' = 1$

chain rule: $\frac{dv}{dx} = v' \times a'$

$= \frac{1}{2}(x-1)^{-1/2}$

$u' = 1 \quad v' = \frac{1}{2}(x-1)^{-1/2}$

product rule $\to$ derivative $= uv' + vu'$

3 marks

$= (x+2) \times \frac{1}{2}(x-1)^{-1/2} + (x-1)^{1/2} \times 1$

$= \frac{x+2}{2\sqrt{x-1}} + \sqrt{x-1} \times \frac{2\sqrt{x-1}}{2\sqrt{x-1}}$

$= \frac{x+2 + 2(x-1)}{2\sqrt{x-1}}$

$= \frac{x+2+2x-2}{2\sqrt{x-1}}$

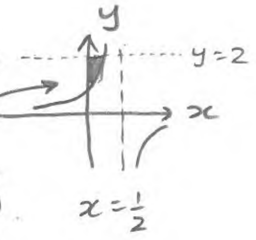
$= \frac{3x}{2\sqrt{x-1}}$

Question 5

Find the area bounded by the curve of f with equation $f(x)=\frac{1}{2-4x}$, the y-axis and the line $y=2$. Write your answer in the form $\frac{a-\log_e(b)}{b}$, where a and b are positive integers.

now $f(x)$ is a hyperbola $\rightarrow$ $f(x) = \frac{1}{2-4x}$

$= \frac{1}{-4(x-\frac{1}{2})}$ $\rightarrow$



we are asked to find this area (shaded)

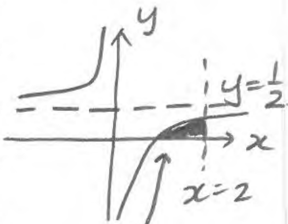
* but it's not between x-terminals

$\rightarrow$ find inverse function $\rightarrow$ $x = \frac{1}{2-4y}$

$\therefore \frac{1}{x} = 2-4y$

$\therefore 2-\frac{1}{x} = 4y$

$\therefore y = \frac{-1}{4x}+\frac{1}{2}$



So you're looking for this area (it's the same area!)

5 marks

$$A = \int_{\frac{1}{2}}^{2}\left(\frac{-1}{4x}+\frac{1}{2}\right)dx$$

$$= \left[\frac{-1}{4}\log_e(x)+\frac{1}{2}x\right]_{\frac{1}{2}}^{2}$$

$$= \left[\frac{-1}{4}\log_e(2)+\frac{1}{2}(2)\right] - \left(\frac{-1}{4}\log_e\left(\frac{1}{2}\right)+\frac{1}{2}\times\frac{1}{2}\right)$$

$$= \frac{-1}{4}\log_e(2) + 1 + \frac{1}{4}\log_e\left(\frac{1}{2}\right) - \frac{1}{4}$$

$$= \frac{-1}{4}\log_e(2) - \frac{1}{4}\log_e(2) - \frac{1}{4} + \frac{4}{4}$$

$$= \frac{-2\log_e(2)+3}{4}$$

$$= \frac{3-2\log_e(2)}{4}$$

$$= \frac{3-\log_e(4)}{4}$$

a. Solve $2 \times 2^{-2x} = 2002$ for x, correct to three decimal places.

b. Simplify, by writing $2\log_e(3x+1) - \log_e(x)$ as a single logarithm expression to base e.

$= \log_e (3x+1)^2 - \log_e(x)$

$= \log_e\left(\frac{(3x+1)^2}{x}\right)$ ✓

1 + 1 = 2 marks

Question 2

I thought this was a minus!

a. Solve the equation $\log_e(3x+5) + \log_e(2) = 2$, for x.

$\therefore \log_e\left(\frac{3x+5}{2}\right) = 2$

$\therefore \frac{3x+5}{2} = e^2$

$\therefore 3x+5 = 2e^2$

$\therefore 3x = 2e^2 - 5$

$\therefore x = \frac{2e^2-5}{3}$

$\log_e(3x+5) + \log_e(2) = 2$

$\therefore \log_e(2(3x+5)) = 2$

$\therefore 2(3x+5) = e^2$

$\therefore 3x+5 = \frac{e^2}{2}$

$\therefore 3x = \frac{e^2}{2} - 5$

$= \frac{e^2-10}{2}$

$\therefore x = \frac{e^2-10}{6}$

2 marks

Question 3

Solve the equation $2\log_e(x-2) - \log_e(x+1) = \log_e(2)$ for x.

$\therefore \log_e(x-2)^2 - \log_e(x+1) = \log_e(2)$

$\therefore \log_e\left(\frac{(x-2)^2}{x+1}\right) = \log_e(2)$

$\therefore \frac{(x-2)^2}{x+1} = 2$

$\therefore (x-2)^2 = 2(x+1)$

$\therefore x^2 - 4x + 4 = 2x + 2$

$\therefore x^2 - 6x + 2 = 0$

so $x = \frac{-(-6) \pm \sqrt{36-8}}{2}$

$= \frac{6 \pm \sqrt{28}}{2}$

$= \frac{6 \pm 2\sqrt{7}}{2}$

$= 3 \pm \sqrt{7}$

3 marks

but $x - 2 > 0$ (from the first expression, $2\log_e(x-2)$)

must be > 0 ✓

$\therefore x = 3 + \sqrt{7}$

The graph of the function with rule $y = \frac{1}{x}$ is transformed as follows:

a dilation by a factor of $\frac{1}{2}$ from the y-axis $\therefore n = 2$ $\quad y = a\,f(n(x-h)) + k$

a reflection in the y-axis $\therefore x' = -x$

a translation of +3 units parallel to the x-axis $h = 3$

a translation of +1 unit parallel to the y-axis $k = 1$

a. Write down the equation of the rule of the transformed function.

$\therefore y' = f(2(-x+3)) + 1$

$= \frac{1}{2(3-x)} + 1$

b. Hence state the domain and range of the transformed function

domain $\therefore 3 - x \neq 0$ $\therefore$ domain is $R\backslash\{3\}$

range is $R\backslash\{1\}$

1 + 2 = 3 marks

Question 2

a. The graph of g is obtained from the graph of the function f with rule $f(x) = x^2$ by a translation by +3 units parallel to the x-axis. Write down the rule for g.

$g(x) = (x-3)^2$

b. The graph of h is obtained from the graph of g by a translation by −1 unit parallel to the y-axis. Write down the rule for h.

$h(x) = x^2 - 1$

c. The graph of k is obtained from the graph of h by a dilation by a scale factor of 0.5 from the y-axis. Write down the rule for k.

$\rightarrow n = 2$ $\quad$ so $k(x) = (2x)^2$

$\therefore k(x) = 4x^2$

1 + 1 + 1 = 3 marks

a. Find the x-coordinates of the points of intersection of the line with equation $y = 3x + 1$ and the parabola with equation $y = 2x^2 + 4x - 5$.

intersection $\therefore 3x+1 = 2x^2+4x-5$

$\therefore \quad 0 = 2x^2 + x - 6$

$x = \dfrac{-1 \pm \sqrt{1+4\times2\times6}}{4} = \dfrac{-1 \pm \sqrt{49}}{4} = \dfrac{-1\pm7}{4} = -2, \dfrac{3}{2}$

when $x = -2$, $y = 3(-2)+1 = -5$

when $x = \frac{3}{2}$, $y = 3\left(\frac{3}{2}\right)+1 = \frac{11}{2}$

$\therefore$ intersections at $(-2,-5)$ and $\left(\frac{3}{2}, \frac{11}{2}\right)$

b. Use calculus to find the area, correct to three decimal places, of the region bounded by the line with equation $y = 3x + 1$ and the parabola with equation $y = 2x^2 + 4x - 5$.

$$\cancel{A = \int_{-2}^{3/2} \left((3x+1) - (2x^2+4x-5)\right) dx}$$

$$= \int_{-2}^{3/2} (-2x^2 - x + 6)\, dx$$

$$= \left[\frac{-2x^3}{3} - \frac{x^2}{2} + 6x\right]_{-2}^{3/2}$$

2 + 3 = 5 marks

$$= \left(-\frac{2}{3}\times\frac{3^3}{2^3} - \frac{1}{2}\times\frac{3^2}{2^2} + 6\times\frac{3}{2}\right) - \left(\frac{-2(-2)^3}{3} - \frac{(-2)^2}{2} + 6(-2)\right)$$

$$= \left(-\frac{3^2}{2^2} - \frac{3^2}{2^3} + 9\right) - \left(\frac{+16}{3} - \frac{4}{2} - 12\right)$$

$$= -\frac{9}{4} - \frac{9}{8} + 9 - \frac{16}{3} + \frac{4}{2} + 12$$

$$= 21 - \frac{16}{3} + \left(\frac{-9}{4}\times\frac{2}{2} - \frac{9}{8} + \frac{4}{2}\times\frac{4}{4}\right)$$

$$= 21 - \frac{16}{3} + \left(\frac{-18}{8} - \frac{9}{8} + \frac{16}{8}\right)$$

$$= 21 - \frac{16}{3} - \frac{11}{8}$$

$$= \frac{343}{24}$$

CALC

$$= 14.292 \text{ units}^2$$

Question 3

Find the exact solutions of the equation $\sin(2\pi x) = -\sqrt{3}\cos(2\pi x)$, $0 \le x \le 1$.

$\therefore \dfrac{\sin(2\pi x)}{\cos(2\pi x)} = -\sqrt{3}$

$\therefore \tan(2\pi x) = -\sqrt{3}$ $\qquad$ $\tan \bigcirc = -\sqrt{3}$ $\therefore \bigcirc$ is in Q2, Q4

Q1 angle is $\pi/3$

$\therefore 2\pi x = 2\pi/3,\ 5\pi/3$ $\leftarrow$ $\quad \therefore \bigcirc = 2\pi/3,\ 5\pi/3$

$\therefore x = 2\pi/3 \div 2\pi,\ \dfrac{5\pi}{3} \div 2\pi$

$\therefore x = \dfrac{1}{3},\ \dfrac{5}{6}$

2 marks

Question 4

For the function $f: [-\pi, \pi] \to R$, $f(x) = 5\cos\left(2\left(x + \frac{\pi}{3}\right)\right)$

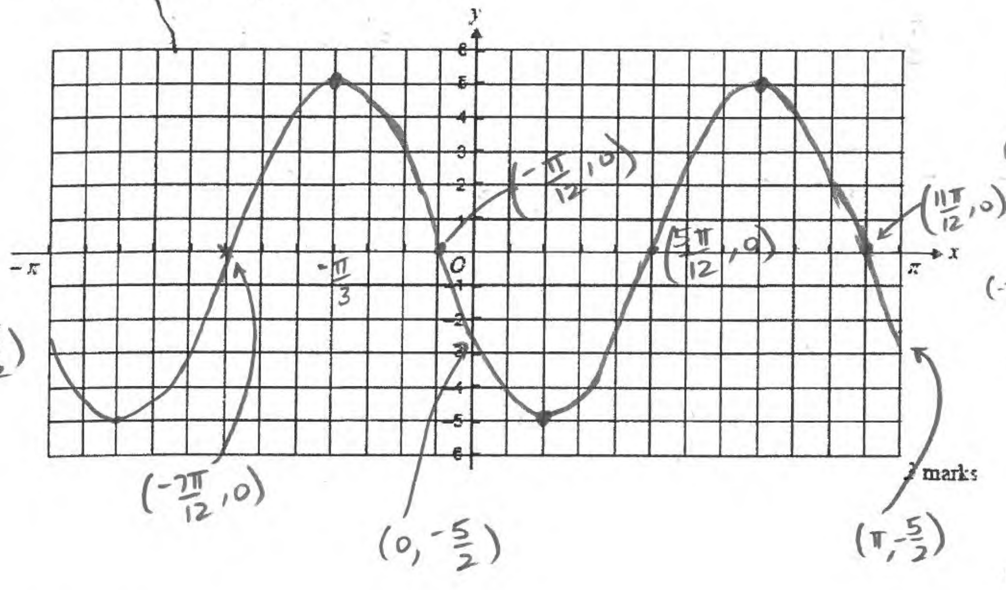
a. write down the amplitude and period of the function

amplitude = 5

period = $\frac{2\pi}{n}$ where $n = 2$ $\therefore$ period = π

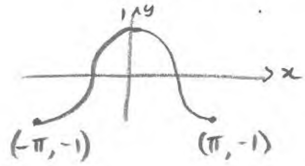
2 marks

b. sketch the graph of the function f on the set of axes below. Label axes intercepts with their coordinates. Label endpoints of the graph with their coordinates.

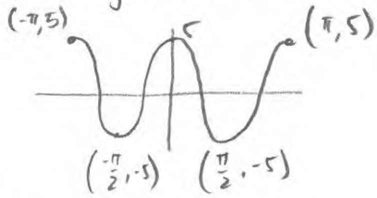


3 marks

$y = \cos(x)$



$y = 5\cos(2x)$



$y = 5\cos(2(x + \frac{\pi}{3}))$

$\leftarrow \frac{\pi}{3}$

end points

$f(-\pi) = 5\cos(2(-\frac{2\pi}{3}))$

$= 5\cos(-\frac{4\pi}{3})$

$= 5(-\frac{1}{2})$

$= -\frac{5}{2}$

$f(\pi) = 5\cos(2(4\pi/3))$

$= 5\cos(8\pi/3)$

$= 5\cos(2\pi/3)$

$= 5(-\frac{1}{2})$

$= -\frac{5}{2}$

Question 8

Let $f: R \rightarrow R,\ f(x) = \sin\left(\frac{2\pi x}{3}\right)$.

a. Solve the equation $\sin\left(\frac{2\pi x}{3}\right) = -\frac{\sqrt{3}}{2}$ for $x \in [0, 3]$.

$\sin = -\sqrt{3}/2$

$\therefore$ is in Q3, Q4

Q1 angle is $\pi/3$

$\therefore = 4\pi/3,\ \frac{5\pi}{3}$

$\therefore \frac{2\pi x}{3} = \frac{4\pi}{3},\ \frac{5\pi}{3}$

$\therefore 2x = 4,\ 5$

$\therefore x = 2,\ \frac{5}{2}$

2 marks

Question 3

Solve the equation $\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\cos = \frac{1}{2}$

$\therefore$ is in Q1, Q4

Q1 angle is $\pi/3$

~~$\therefore$~~ $= \frac{\pi}{3},\ \frac{5\pi}{3}$

and period of this function is $\frac{2\pi}{n}$ where

$n = \frac{3}{2}$ $\quad \therefore p = 2\pi \div \frac{3}{2}$

$= \frac{4\pi}{3}$

$\therefore \frac{3x}{2} = \frac{\pi}{3},\ \frac{5\pi}{3}$

$\therefore 3x = \frac{2\pi}{3},\ \frac{10\pi}{3}$ too big

$\therefore x = \frac{2\pi}{9},\ \cancel{\frac{10\pi}{9}},\ \frac{10\pi}{9}$ - period

$= \frac{2\pi}{9},\ \frac{10\pi}{9} - \frac{4\pi}{3}$

$= \frac{2\pi}{9},\ \frac{10\pi - 12\pi}{9}$

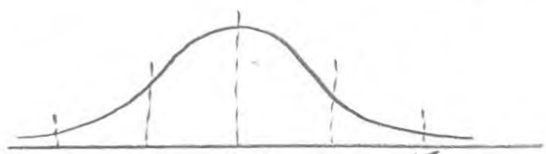
$\boxed{x = \frac{2\pi}{9},\ \frac{-2\pi}{9}}$

2 marks

Question 1

The diameters of circular mats produced by a machine are normally distributed. with mean 12 cm and standard deviation 1.5 cm.

a. Sketch the normal distribution curve for the diameters of the circular mats produced by the machine.



b. It is known that exactly 16.00 % of mats produced by the machine have a diameter less than k cm. Find the value of k, correct to one decimal place.

if tech free, $x = 10 \cdot 5$ cm

2 + 1 = 3 marks

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{12} & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

a. Find Pr $(X < 3)$.

$$Pr(X<3) = \int_1^3 \frac{x}{12}\,dx$$

$$= \left[\frac{1}{2}\frac{x^2}{12}\right]_1^3$$

$$= \frac{9}{24} - \frac{1}{24}$$

$$= \frac{8}{24}$$

$$= \frac{1}{3}$$

2 marks

b. If Pr $(X \ge a) = \frac{5}{8}$, find the value of a.

$$\frac{5}{8} = \int_a^5 \frac{x}{12}\,dx$$

$$= \left[\frac{x^2}{24}\right]_a^5$$

$$\therefore \frac{5}{8} = \frac{5^2}{24} - \frac{a^2}{24}$$

$$\therefore \frac{15}{24} = \frac{25}{24} - \frac{a^2}{24}$$

$$\therefore a^2 = 10$$

2 marks

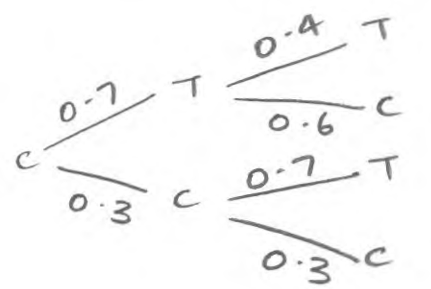
$\therefore a = \sqrt{10}$ (not $-\sqrt{10}$ as $1 < x < 5$)

Question 10

Jo has either tea or coffee at morning break. If she has tea one morning, the probability she has tea the next morning is 0.4. If she has coffee one morning, the probability she has coffee the next morning is 0.3. Suppose she has coffee on a Monday morning. What is the probability that she has tea on the following Wednesday morning?

$$\Pr(T \text{ on Wed}) = \Pr(TT) + \Pr(CT)$$
$$= 0.7 \times 0.4 + 0.3 \times 0.7$$
$$= 0.28 + 0.21$$
$$= 0.49$$

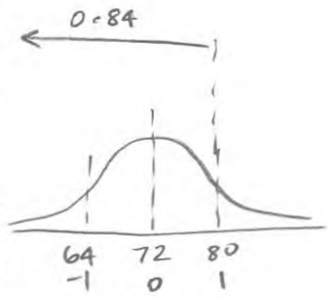
3 marks



Question 5

Let X be a normally distributed random variable with a mean of 72 and a standard deviation of 8. Let Z be the standard normal random variable. Use the result that $\Pr(Z < 1) = 0.84$, **correct to two decimal places**, to find

a. the probability that X is greater than 80



$\Pr(X > 80) = 1 - \Pr(X < 80)$

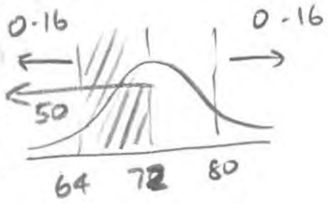
$= 1 - 0.84$

$= 0.16$

1 mark

b. the probability that $64 < X < 72$

desired area is shaded (right)



$= 0.50 - 0.16$

$= 0.34$

$\therefore \Pr(64 < X < 72) = 0.34$

1 mark

c. the probability that $X < 64$ given that $X < 72$.

$\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$ $\quad A \to X < 64$, $\quad B \to X < 72$

$\Pr(A \cap B) = \Pr(X < 64)$ $\quad \Pr(X < 72) = \Pr(B)$

$= 0.16$ $\quad = 0.5$

$\therefore \Pr(A|B) = \dfrac{0.16}{0.5}$

$= 0.32$

$\therefore \Pr(X < 64 | X < 72) = 0.32$

2 marks

Question 5

It is known that 50% of the customers who enter a restaurant order a cup of coffee. If four customers enter the restaurant, what is the probability that more than two of these customers order coffee? (Assume that what any customer orders is independent of what any other customer orders.)

more than 2 $\to$ $\Pr(X = 3) + \Pr(X = 4)$

$\Pr(X = 3) = \binom{4}{3}(0.5)^3(0.5)$

$= 4 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)$

$= \frac{1}{4}$

$\Pr(X = 4) = \left(\frac{1}{2}\right)^4$

$= \frac{1}{16}$

2 marks

$\therefore$ Pr (more than 2 customers order coffee)

$= \frac{1}{4} + \frac{1}{16}$

$= \frac{5}{16}$

4

Question 6

Two events, A and B, from a given event space, are such that $\Pr(A) = \frac{1}{5}$ and $\Pr(B) = \frac{1}{3}$.

a. Calculate $\Pr(A' \cap B)$ when $\Pr(A \cap B) = \frac{1}{8}$.

	A	A'	
B	$\frac{1}{8}$	$\frac{1}{3} - \frac{1}{8}$	$\frac{1}{3}$
B'	$\frac{1}{5} - \frac{1}{8}$		
	$\frac{1}{5}$		1

$$\Pr(A' \cap B) = \frac{1}{3} - \frac{1}{8}$$
$$= \frac{8}{24} - \frac{3}{24}$$
$$= \frac{5}{24}$$

1 mark

b. Calculate $\Pr(A' \cap B)$ when A and B are mutually exclusive events.

mutually exclusive

$\therefore \Pr(A \cap B) = 0$

$\therefore \Pr(A' \cap B) = \frac{1}{3}$

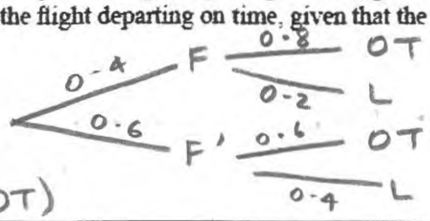
1 mark

Question 11

There is a daily flight from Paradise Island to Melbourne. The probability of the flight departing on time, given that there is fine weather on the island, is 0.8, and the probability of the flight departing on time, given that the weather on the island is not fine, is 0.6.

In March the probability of a day being fine is 0.4.

Find the probability that on a particular day in March



a. the flight from Paradise Island departs on time

$$\Pr(OT) = \Pr(F \cap OT) + \Pr(F' \cap OT)$$
$$= 0.4 \times 0.8 + 0.6 \times 0.6$$
$$= 0.32 + 0.36$$
$$= 0.68$$

2 marks

b. the weather is fine on Paradise Island, given that the flight departs on time.

$$\Pr(F \mid OT) = \frac{\Pr(F \cap OT)}{\Pr(OT)}$$

2 marks

$$= \frac{0.32}{0.68}$$
$$= \frac{0.08}{0.17}$$
$$= \frac{8}{17}$$

Question 4

The function

$$f(x) = \begin{cases} k\sin(\pi x) & \text{if } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for the continuous random variable X.

a. Show that $k = \frac{\pi}{2}$.

For a probability density function, $\int f(x) = 1$

now $\int_0^1 k\sin(\pi x)\,dx = \left[-\frac{k}{\pi}\cos(\pi x)\right]_0^1$

$\therefore 1 = -\frac{k}{\pi}\cos(\pi) - -\frac{k}{\pi}\cos(0)$

$= -\frac{k}{\pi}(-1) + \frac{k}{\pi}(1)$

$= \frac{2k}{\pi}$

$\therefore \pi = 2k$

$\therefore k = \frac{\pi}{2}$ as required

b. Find $\Pr\left(X \le \frac{1}{4} \mid X \le \frac{1}{2}\right)$.

$\Pr(X \le \frac{1}{4} \mid X \le \frac{1}{2}) = \frac{\Pr(X \le \frac{1}{4} \cap X \le \frac{1}{2})}{\Pr(X \le \frac{1}{2})}$

$= \frac{\Pr(X \le \frac{1}{4})}{\Pr(X \le \frac{1}{2})}$

$\Pr(X \le \frac{1}{4}) = \int_0^{\frac{1}{4}} \frac{\pi}{2}\sin(\pi x)\,dx$

$= \left[-\frac{1}{2}\cos(\pi x)\right]_0^{1/4}$

$= -\frac{1}{2}\cos\left(\frac{\pi}{4}\right) - -\frac{1}{2}\cos(\pi)$

$= -\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$

$= \frac{1}{2} - \frac{1}{2\sqrt{2}}$

$\Pr(X \le \frac{1}{2}) = \left[-\frac{1}{2}\cos(\pi x)\right]_0^{1/2}$

$= -\frac{1}{2}\cos\left(\frac{\pi}{2}\right) - -\frac{1}{2}\cos(0)$

$= 0 + \frac{1}{2}$

$= \frac{1}{2}$

so $\frac{\Pr(X \le \frac{1}{4})}{\Pr(X < \frac{1}{2})} = \frac{\frac{1}{2} - \frac{1}{2\sqrt{2}}}{\frac{1}{2}}$

2 + 3 = 5 marks

$= 2\left(\frac{1}{2} - \frac{1}{2\sqrt{2}}\right)$

$= 1 - \frac{1}{\sqrt{2}}$

$\therefore \Pr(X \le \frac{1}{4} \mid X \le \frac{1}{2}) = \frac{\sqrt{2} - 1}{\sqrt{2}}$

Question 7

Jane drives to work each morning and passes through three intersections with traffic lights. The number X of traffic lights that are red when Jane is driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.1	0.2	0.3	0.4

a. What is the mode of X?

Mode → highest $Pr(x=x)$'s x value

$\therefore$ Mode = 4

b. Jane drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days?

$Pr(\text{same}) = Pr(0,0) + Pr(1,1) + Pr(2,2) + Pr(3,3)$

$= 0.1 \times 0.1 + 0.2 \times 0.2 + 0.3 \times 0.3 + 0.4 \times 0.4$

$= 0.01 + 0.04 + 0.09 + 0.16$

$= 0.3$

1 + 2 = 3 marks

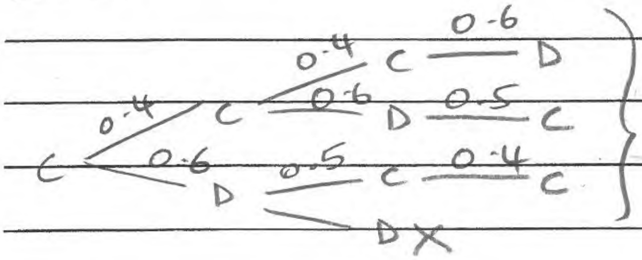
Question 8

Every Friday Jean-Paul goes to see a movie. He always goes to one of two local cinemas – the Dandy or the Cino.

If he goes to the Dandy one Friday, the probability that he goes to the Cino the next Friday is 0.5. If he goes to the Cino one Friday, then the probability that he goes to the Dandy the next Friday is 0.6.

On any given Friday the cinema he goes to depends only on the cinema he went to on the previous Friday.

If he goes to the Cino one Friday, what is the probability that he goes to the Cino on exactly two of the next three Fridays?



$Pr(C \text{ on exactly 2 occasions})$

$= Pr(CCD) + Pr(CDC) + Pr(DCC)$

$Pr(CCD) = 0.4 \times 0.4 \times 0.6 = 0.096$

$Pr(CDC) = 0.4 \times 0.6 \times 0.5 = 0.12$

$Pr(DCC) = 0.6 \times 0.5 \times 0.4 = 0.12$

$\therefore Pr(C \text{ exactly twice}) = 0.096 + 0.12 + 0.12$

$= 0.336$

3 marks

Question 9

If $X \sim \text{Bi}(5, 0.1)$, find $\Pr(X = 3)$.

$n = 5 \qquad \therefore \Pr(x=3) = \binom{5}{3}(0.1)^3(0.9)^2$

$p = 0.1 \qquad = \frac{5!}{3!\,2!}\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^2$

$x = 3 \qquad = \frac{5 \times 4}{2}\left(\frac{1}{10^3}\right)\left(\frac{81}{10^2}\right)$

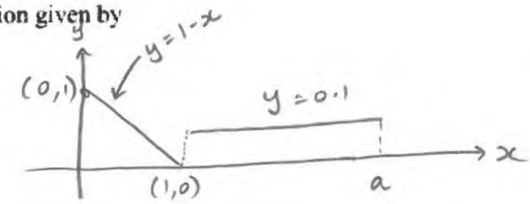
$= 10 \times \frac{81}{10^5} = \frac{81}{10^4} = \frac{81}{10000}$

2 marks

Question 10

The probability density function of a random variable X has a density function given by

$$1-x \longrightarrow \quad f(x) = \begin{cases} \cancel{} & 0 \le x \le 1 \\ 0.1 & 1 < x \le a \\ 0 & \text{elsewhere} \end{cases}$$



a. Find the value of a.

$\int f(x)\,dx = 1$

now area under $y = 1-x,\ 0 \le x \le 1 = \frac{1}{2}bh = \frac{1}{2}$

$\therefore$ area under $y = 0.1 = 1 - \frac{1}{2} = \frac{1}{2}$

$= (a-1) \times 0.1$

$\therefore \frac{1}{2} = 0.1a - 0.1$

1 mark

$\therefore 0.6 = 0.1 \times a \qquad \therefore a = 6$

b. Find $\Pr(X > 0.5)$.

$\Pr(x > 0.5) = \Pr(0.5 < x < 1) + \Pr(\text{rectangular area})$

$= \frac{1}{2} \times bh + \frac{1}{2}$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2}$

1 mark

$= \frac{5}{8}$

c. Find $\Pr(X > 2 \mid X > 0.5)$.

$\Pr(x > 2 \mid x > 0.5) = \frac{\Pr(x > 2 \cap x > 0.5)}{\Pr(x > 0.5)} = \frac{\Pr(x > 2)}{\Pr(x > 0.5)}$

Now $\Pr(x > 2) = 0.4$ (by observation) $= \frac{4}{10}$

$\therefore \Pr(x > 2 \mid x > 0.5) = \frac{4}{10} \div \frac{5}{8}$

$= \frac{\cancel{4}}{{}_5\cancel{10}} \times \frac{\cancel{8}^{\,4}}{5}$

$= \frac{16}{25}$

2 marks