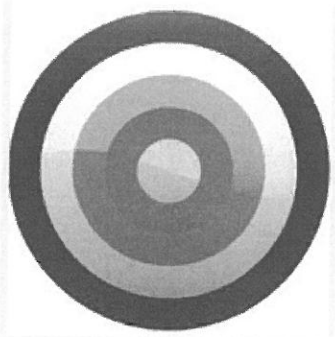
12MME

"MUST-KNOW" QUESTIONS

Target  100%

Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET

Question 3

a. Let $f(x) = e^{\cos(x)}$. Find $f'(x)$

1 mark

b. Let $y = x\tan(x)$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$.

3 marks

Question 1

Let $f(x) = \frac{x^3}{\sin(x)}$. Find $f'(x)$.

2 marks

b. Let $g(x) = \log_e(\tan(x))$. Evaluate $g'\left(\frac{\pi}{4}\right)$.

2 marks

Question 1

a. Let $y = (3x^2 - 5x)^5$. Find $\frac{dy}{dx}$.

b. Let $f(x) = xe^{3x}$. Evaluate $f'(0)$.

2 + 3 = 5 marks

Question 4

a. Differentiate $(x+2)\sqrt{(x-1)}$, giving your answer as a single fraction.

3 marks

Question 5

Find the area bounded by the curve of f with equation $f(x) = \frac{1}{2-4x}$, the y-axis and the line $y = 2$. Write your answer in the form $\frac{a - \log_e(b)}{b}$, where a and b are positive integers.

5 marks

a. Solve $2 \times 2^{-2x} = 2002$ for x, correct to three decimal places.

b. Simplify, by writing $2\log_e(3x+1) - \log_e(x)$ as a single logarithm expression to base e.

1 + 1 = 2 marks

Question 2

a. Solve the equation $\log_e(3x+5) + \log_e(2) = 2$, for x.

2 marks

Question 3

Solve the equation $2\log_e(x-2) - \log_e(x+1) = \log_e(2)$ for x.

3 marks

The graph of the function with rule $y = \frac{1}{x}$ is transformed as follows:

a dilation by a factor of $\frac{1}{2}$ from the y-axis

a reflection in the y-axis

a translation of $+3$ units parallel to the x-axis

a translation of $+1$ unit parallel to the y-axis.

a. Write down the equation of the rule of the transformed function.

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_____

b. Hence state the domain and range of the transformed function.

_____

_____

1 + 2 = 3 marks

Question 2

a. The graph of g is obtained from the graph of the function f with rule $f(x) = x^2$ by a translation by $+3$ units parallel to the x-axis. Write down the rule for g.

_____

_____

b. The graph of h is obtained from the graph of g by a translation by -1 unit parallel to the y-axis. Write down the rule for h.

_____

_____

c. The graph of k is obtained from the graph of h by a dilation by a scale factor of 0.5 from the y-axis. Write down the rule for k.

_____

_____

1 + 1 + 1 = 3 marks

a. Find the x-**coordinates** of the points of intersection of the line with equation $y = 3x + 1$ and the parabola with equation $y = 2x^2 + 4x - 5$.

b. Use calculus to find the area, correct to three decimal places, of the region bounded by the line with equation $y = 3x + 1$ and the parabola with equation $y = 2x^2 + 4x - 5$.

2 + 3 = 5 marks

Question 3

Find the **exact** solutions of the equation $\sin(2\pi x) = -\sqrt{3}\cos(2\pi x)$, $0 \le x \le 1$.

2 marks

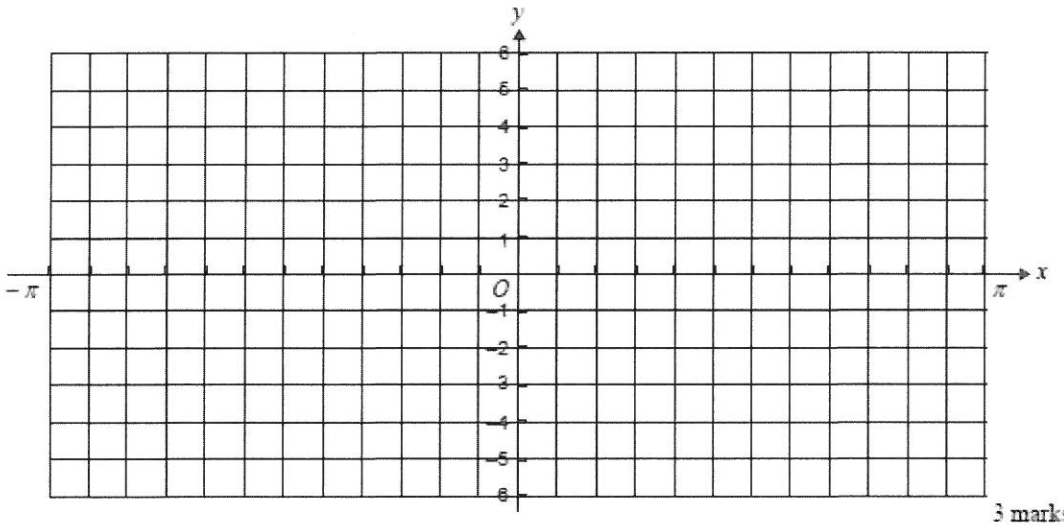
Question 4

For the function $f:[-\pi, \pi] \to R$, $f(x) = 5\cos\left(2\left(x + \frac{\pi}{3}\right)\right)$

a. write down the amplitude and period of the function

2 marks

b. sketch the graph of the function f on the set of axes below. Label axes intercepts with their coordinates. Label endpoints of the graph with their coordinates.



3 marks

Question 8

Let $f: R \to R,\ f(x) = \sin\left(\frac{2\pi x}{3}\right)$.

a. Solve the equation $\sin\left(\frac{2\pi x}{3}\right) = -\frac{\sqrt{3}}{2}$ for $x \in [0, 3]$.

2 marks

Question 3

Solve the equation $\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

2 marks

Question 4

A wine glass is being filled with wine at a rate of $8\,\text{cm}^3/\text{s}$. The volume, $V\,\text{cm}^3$, of wine in the glass when the depth of wine in the glass is x cm is given by $V = 4x^{\frac{3}{2}}$. Find the rate at which the depth of wine in the glass is increasing when the depth is 4 cm.

3 marks

PROBABILITY

Question 1

The diameters of circular mats produced by a machine are normally distributed. with mean 12 cm and standard deviation 1.5 cm.

a. Sketch the normal distribution curve for the diameters of the circular mats produced by the machine.

b. It is known that exactly 16.00 % of mats produced by the machine have a diameter less than k cm. Find the value of k, correct to one decimal place.

2 + 1 = 3 marks

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{12} & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

a. Find $\Pr(X < 3)$.

2 marks

b. If $\Pr(X \ge a) = \frac{5}{8}$, find the value of a.

2 marks

Question 10

Jo has either tea or coffee at morning break. If she has tea one morning, the probability she has tea the next morning is 0.4. If she has coffee one morning, the probability she has coffee the next morning is 0.3. Suppose she has coffee on a Monday morning. What is the probability that she has tea on the following Wednesday morning?

3 marks

Question 5

Let X be a normally distributed random variable with a mean of 72 and a standard deviation of 8. Let Z be the standard normal random variable. **Use the result that $\Pr(Z < 1) = 0.84$, correct to two decimal places**, to find

a. the probability that X is greater than 80

1 mark

b. the probability that $64 < X < 72$

1 mark

c. the probability that $X < 64$ given that $X < 72$.

2 marks

Question 5

It is known that 50% of the customers who enter a restaurant order a cup of coffee. If four customers enter the restaurant, what is the probability that more than two of these customers order coffee? (Assume that what any customer orders is independent of what any other customer orders.)

2 marks

Question 6

Two events, A and B, from a given event space, are such that $\Pr(A) = \frac{1}{5}$ and $\Pr(B) = \frac{1}{3}$.

a. Calculate $\Pr(A' \cap B)$ when $\Pr(A \cap B) = \frac{1}{8}$.

1 mark

b. Calculate $\Pr(A' \cap B)$ when A and B are mutually exclusive events.

1 mark

Question 11

There is a daily flight from Paradise Island to Melbourne. The probability of the flight departing on time, given that there is fine weather on the island, is 0.8, and the probability of the flight departing on time, given that the weather on the island is not fine, is 0.6.

In March the probability of a day being fine is 0.4.

Find the probability that on a particular day in March

a. the flight from Paradise Island departs on time

2 marks

b. the weather is fine on Paradise Island, given that the flight departs on time.

2 marks

Question 4

The function

$$f(x) = \begin{cases} k\sin(\pi x) & \text{if } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for the continuous random variable X.

a. Show that $k = \frac{\pi}{2}$.

b. Find $\Pr\left(X \leq \frac{1}{4} \mid X \leq \frac{1}{2}\right)$.

2 + 3 = 5 marks

Question 7

Jane drives to work each morning and passes through three intersections with traffic lights. The number X of traffic lights that are red when Jane is driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.1	0.2	0.3	0.4

a. What is the mode of X?

b. Jane drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days?

1 + 2 = 3 marks

Question 8

Every Friday Jean-Paul goes to see a movie. He always goes to one of two local cinemas – the Dandy or the Cino.

If he goes to the Dandy one Friday, the probability that he goes to the Cino the next Friday is 0.5. If he goes to the Cino one Friday, then the probability that he goes to the Dandy the next Friday is 0.6.

On any given Friday the cinema he goes to depends only on the cinema he went to on the previous Friday.

If he goes to the Cino one Friday, what is the probability that he goes to the Cino on **exactly two** of the next three Fridays?

3 marks

Question 9

If $X \sim \text{Bi}(5, 0.1)$, find $\Pr(X = 3)$.

2 marks

Question 10

The probability density function of a random variable X has a density function given by

$$f(x) = \begin{cases} |x-1| & 0 \le x \le 1 \\ 0.1 & 1 < x \le a \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the value of a.

1 mark

b. Find $\Pr(X > 0.5)$.

1 mark

c. Find $\Pr(X > 2 \mid X > 0.5)$.

2 marks